Heat Transfer to Cylinders in a Confined Jet at High Temperature

P. H. CHEVALIER, G. R. KUBANEK, and W. H. GAUVIN

McGill University and Pulp and Paper Research Institute of Canada, Montreal, Quebec

Temperature and velocity measurements were carried out at the center of the test section of an 8-in. I.D. cylindrical graphite chamber in a confined nitrogen jet issuing from a 40-kw. D.C. plasma torch. The test section was located 10.5 in. below the torch nozzle, and temperatures and velocities up to 4,300°F. and 120 ft./sec. were obtained.

The overall rates of heat transfer by forced convection were measured to water cooled circular and square cylinders in two orientations. Results indicated that, owing to the high level of turbulence of the jet, turbulent boundary layers prevailed in the low range of Reynolds numbers investigated. Correlations are presented in terms of the Nusselt, Reynolds, and Prandtl numbers, with the physical properties of the gas evaluated at the bulk temperature.

Research in the field of high temperature chemical and metallurgical processing is rapidly gaining momentum, principally as a result of recent progress in the design of direct current plasma torches.

Kubanek et al. (*1*) studied the pure convective heat transfer from a nitrogen plasma jet to water cooled stationary spheres. Their results showed that in the low range of Reynolds numbers investigated (630 to 4,300), the boundary layer was turbulent in nature.

The purpose of the present work was to study, under similar conditions, the behavior of circular cylinders and of a cylinder with a square cross section in two orientations.

HEAT TRANSFER TO CYLINDERS

Heat transfer by forced convection to cylinders in crossflow has been extensively investigated in the case of circular cylinders (*2* to *19*), whereas an almost negligible number of studies has dealt with other cross-sectional geometries, such as elliptic (*9, 20, 21*), square (*16, 20*), rectangular (*22*), or hexagonal (*16*). All these studies have been conducted at low or moderate temperature differences (less than 350°F.) and in streams of relatively low turbulence levels. For circular cylinders, however, a few experiments (*16, 23* to *30*) were carried out with temperature differences ranging from 1,000° to 5,000°F. In some of them (*16, 29, 30*), the wall was raised to elevated temperature (1,000° to 1,800°F.), while the gas was at room temperature. Hilpert (*16*) observed a 6% increase in hD/k_m, as the surface temperature was increased from 200° to 1,800°F.

A considerable divergence of opinion exists concerning the temperature basis at which the physical properties must be evaluated. Some authors (*1, 25, 31* to *33*) recommend the bulk temperature, others the film temperature T_f defined as $[0.5\ (T_b + T_w)]$ (*3, 14, 34, 35*) or as $[0.5\ (T_{aw} + T_w)]$ by Talmor (*36*). Kestin and Maeder (*5*) and Hilpert (*16*) integrated the physical properties through the boundary layer.

The effect of the intensity of turbulence on the local heat transfer coefficient and on the average coefficient has been investigated (*2, 4, 8, 9, 37*) and (*5, 12, 14*), respectively. Sogin and Subramanian (*38*) studied its effects on the mass transfer coefficient. Although a considerable disagreement exists between the results of the various workers, general conclusions may be summarized as follows: free-stream turbulence affects the heat (and mass) transfer rate both locally and through flow configuration, and the local effect is much greater on a laminar than on a turbulent boundary layer or on the wake.

Van der Hegge Zijnen (*14*) seems to be the only one who made a systematic investigation of the combined influence on the scale and intensity of turbulence. Although his experiments were carefully conducted, it should be noted that his data were quite scattered and that some of his conclusions about the effect of scale of turbulence were opposed to those predicted by the Taylor theory (*39*).

It has been shown that turbulence causes transition in the boundary layer to occur at a much lower critical Reynolds number. While Taylor (*39*) and Wieghardt (*40*) demonstrated that N_{Re_c} depends on the so-called *Taylor parameter* $\Lambda = I\ (L_x/D)^{0.2}$ for isotropic and low turbulence streams, Torobin and Gauvin (*41*), in their experimental study on freely moving spheres at low N_{Re} (400 to 3,000) and high relative intensity of turbulence (1 to

P. H. Chevalier is with La Shell Francaise, Rouen, France; G. R. Kubanek is with Johnson and Johnson Research, New Brunswick, New Jersey; W. H. Gauvin is with Noranda Research Centre, Pointe Claire, Quebec.

40%), found that I was the first causative factor of transition and obtained a criterion predicting transition:

$$I^2 \, N_{Re_c} = \text{constant} = 45 \text{ for spheres} \quad (1)$$

Van Driest and Blumer (*42*) proposed an equation which predicts, at transition, the location of the transition point at the surface. Recently, Talmor (*36*) applied the equation in reverse to obtain the turbulence intensity in a combustion induced turbulent stream from a knowledge of the point of transition. As the calculated intensity was of the expected magnitude, Talmor proposed that this method may provide a useful means of indirect intensity determination in situations where direct measurements are impossible.

It is worthwhile mentioning the hydrodynamic and heat transfer investigation carried out by Harlow and Fromm (*22*) in the von Kármán wake of a rectangular cylinder at low temperature differences and in the N_{Re} range of 25 to 400. Their results for heat transfer, corrected for blockage effects, showed that for $N_{Re} < 100$, N_{Nu} was proportional to N_{Re} to the ½ power, whereas for $N_{Re} > 150$, N_{Nu} was proportional to N_{Re} to the 2/3 power. The latter N_{Nu}-N_{Re} dependency is in agreement with the universal behavior for heat transfer in the region of separated flow suggested by Richardson (*13*).

Several characteristic lengths have been proposed to predict convective heat (and mass) transfer to various particles from a standard relationship. Krischer and Loos' characteristic length L' (*43*) was found to be applicable to two-dimensional bodies only. Pasternak and Gauvin (*44*) studied convective heat and mass transfer for twenty shapes suspended in various orientations in a hot turbulent air stream with a turbulent intensity of 10% at moderate N_{Re} of 400 to 8,000. They derived a characteristic length L'' from qualitative boundary-layer considerations which took into account the body shape and the orientation of the particle and which was defined as

$$L'' = A/p_m \quad (2)$$

Shchitnikov (*45*) tried to apply the characteristic length of Pasternak and Gauvin and that of Polonskaya and Melnikova (*46*) (defined as $A^{0.5}$) to the case of pure convective heat transfer at much higher N_{Re} (10,000 to 140,000) and found a better correlation of his data by using the reduced perimeter over the midbody cross section (p_m/π).

Relationships to predict forced convective heat transfer to circular cylinders for low and moderate temperature differences are abundant. They can be obtained in references *3, 10, 13, 14, 16, 17, 19,* and *35* and *2, 18,* and *35* for gases and liquids, respectively. The heat transfer investigations at high temperatures, which are particularly pertinent to this work, will now be examined.

Scadron and Warshawsky (*27*) have studied heat transfer to bare thermocouples by considering the hot junction as consisting of two cylinders. They obtained

$$N_{Nu_t} = 0.478 \; N_{Re_t}^{0.5} \; N_{Pr_t}^{0.3} \quad (3)$$

in the ranges $250 < N_{Re_t} < 30{,}000$ and $0.1 < N_M < 0.9$, with the gas properties taken at the total temperature. This relationship has been verified by Glawe and Johnson (*47*) at temperatures up to 3,000°F.

Churchill and Brier (*25*) investigated the local heat transfer coefficient at low N_{Re}, ($300 < N_{Re_b} < 2{,}300$) and relatively high temperature differences ($580 < \Delta T < 1{,}800$°F.) in a low turbulence level nitrogen stream ($I <$ 2%). Their average coefficients were 30% higher than those predicted by McAdams' correlation. They proposed the following equation:

$$N_{Nu_b} = 0.60 \; N_{Re_b}^{0.5} \; N_{Pr_b}^{0.33} \; (T_b/T_w)^{0.12} \quad (4)$$

Douglas and Churchill (*34*) replotted the data available for both the cooling and heating of a cylinder by a gas and showed that a single $N_{Nu} - N_{Re}$ curve could be obtained if all the properties were evaluated at T_f. A semiempirical equation was presented

$$N_{Nu_f} = 0.46 \; N_{Re_f}^{0.5} + 0.00128 \; N_{Re_f} \quad (5)$$

in the range $500 < N_{Re_f} < 300{,}000$.

Chludzinski et al. (*26*) measured the heat transfer to a thermocouple immersed for periods less than 0.1 sec. in a 5-kw., radio-frequency, argon-nitrogen plasma jet. The coefficients they obtained were more than twice as great as those predicted by Scadron and Warshawsky's equation, owing to atom recombination at the surface, and were correlated by the following correlation:

$$\begin{aligned} h = {} & 1.05 \; (k_b/D) \; N_{Re_b}^{0.5} \; N_{Pr_b}^{0.3} \\ & + (0.20/12D)^{0.5} \, [1.72 \; 10^{-20} \; C_{A^+} \\ & + 1.11 \; 10^{-20} \; C_{N^+} + 7.5 \; 10^{-20} \; C_N] \end{aligned} \quad (6)$$

The first term in Equation (6) is the convective contribution, while the second is the reaction contribution. It is worthy of mention that the gas velocity was not measured, and an average value, based on the total flow rate, was used for calculating N_{Re} which ranged from 2 to 10.

Recently, Talmor (*36*) reported heat transfer data for cylinders immersed in a high temperature (5,000°F.), high blockage ratio ($D/W = 0.75$), transonic stream which consisted of combustion products. Under these conditions, the boundary layer was found to be essentially turbulent, transition taking place at about 10 deg. from the front stagnation point.

Kubanek et al. (*1*) studied the heat transfer to spheres in a confined nitrogen plasma jet. In the N_{Re} range covered (600 to 4,300), with temperatures up to 5,000°F., the boundary layer was found to be turbulent as indicated by the value close to 0.8 of the exponent of the Reynolds number. The correlation proposed was:
For spheres:

$$N_{Nu_b} = 0.118 \; N_{Re_b}^{0.76} \; N_{Pr_b}^{0.33} \quad (7)$$

EXPERIMENTAL APPARATUS

The equipment used in the present investigation has been described in detail in reference 48, and only its main features are presented here.

The nitrogen stream was heated to high temperature by a direct current plasma torch. The plasma generating system consisted of four units: the plasma torch, the control console, the power supply, and the cooling system. Two sizes of nozzles were employed (with the corresponding matching cathodes): No. 1 with a diameter of 0.219 in. and No. 3 with a diameter of 0.312 in.

An important feature of the direct current arc is the intermittent nature of the arc process near the anode. The sheath of cool gas is bridged electrically by a moving concentrated arc, which leads to a cyclic variation of the voltage between cathode and anode at essentially constant current. Reignition of the arc after each cycle of travel down the nozzle occurs by breakdown of the gas in the neighbourhood of the cathode. Wheaton and Dean (*49*) have reported that this cyclic process has a frequency of about 10,000 cycles/sec. in a torch operating with nitrogen. This induces on the efflux some nonhomogeneities known as *pockets of plasma,* representing regions of high local temperature. These have observed in an argon plasma jet by Watson et al. (*50*).

The plasma jet was fed vertically downward through an 8.0- and 10.0-in. O.D. type AGR graphite tube, 71.75 in. long, at atmospheric pressure. The top and bottom of the chamber

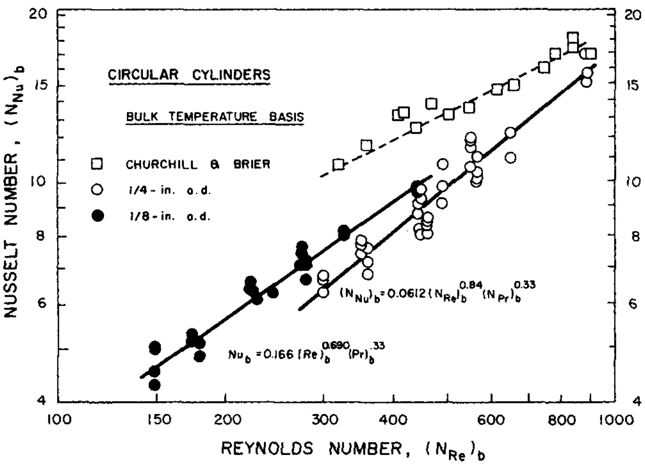

Fig. 1. Heat transfer results for ¼- and ⅛-in. O.D. circular cylinders. Properties at bulk temperature.

were closed by means of cooling plates, and the graphite was protected from ambient air by 12 in. of Fiberfrax insulating material. Access to the chamber test section, located at 10.5 in. from the nozzle, was achieved by means of two diametrically opposite ports allowing insertion of probes or cylinders. Each port had an inside diameter of 1.25 in. and a wall thickness of 0.625 in. and was press fitted into the graphite chamber, with the ends matching the inside chamber curvature.

TECHNIQUE OF MEASUREMENTS

Temperature and Velocity Measurements

The nitrogen temperature profile at the level of the test section was measured by means of bare wire, 0.005-in. diameter tungsten, 5% rhenium/tungsten, 26% rhenium thermocouples. The sheath and insulation materials were inconel and magnesia, respectively, with an outside diameter of 0.040 in. The thermocouple was enclosed inside a water cooled ¼-in. I.D. brass tube with a conical end, out of which the bare tip was made to protrude while temperature measurements were being obtained. Extension lead wire connected the thermocouple to an ice bath and to a variable-zero and variable-span strip chart recorder, whose range could be adjusted from 0 to 5 to 0 to 55 mv. The thermocouple characteristics were provided by the manufacturer and confirmed in reference *62*.

The velocity was obtained by means of a modified, water cooled aspirating probe. The modifications, which converted the aspirating probe to a total heat probe, were made by soldering a 0.064-, 0.023-in. I.D. and ⅜ in. long type 316 stainless steel tube to the opening. The water cooled tube bundle to which the tip was joined prevented it from melting, although a surface layer of scale was sometimes formed as the end glowed white hot. A water cooled 0.75-in. O.D. sleeve, extending to within 6 in. of the tip, provided additional cooling for the probe. A reference pressure, close to the static pressure in value, was measured inside the port, and the uncorrected dynamic pressure was obtained from an inclined manometer, with methanol as working fluid. Inclinations of 1/10 and 1/25 were used. Fluctuations in the pressures were damped out by means of short lengths of 0.010-in. I.D. capillary tubing. Because of the recirculation present in the region of the chamber near the wall, the probe had to be inverted to obtain the total pressure of this upward flow. It was found that the pressure measured by the probe pointing upwards with the flow near the wall was very slightly lower than the reference pressure in value, so this was taken as the static pressure and a corresponding correction was applied to the dynamic pressure readings.

Because of the high gas temperature involved, the method of Scadron and Warshawsky (*27*) was used to calculate the radiation and conduction corrections for the bare wire thermocouples. These corrections for the thermocouple readings as well as the velocity determinations were obtained by an iterative procedure described in detail in reference *48*.

Complete gas velocity and temperature profiles across the reactor at various levels can also be found in reference *48*.

Heat Transfer Measurements

The overall heat transfer from the gas to the water cooled cylinders was obtained by measuring the water temperature rise between two cross sections of brass, thin walled cylinders (called *in* and *out* and symmetrically spaced at 0.500-in. on each side of the jet axis) for a given water flow rate. The latter was simply determined by time volumetry:

$$Q = M\ c_p\ (T_{\text{out}} - T_{\text{in}}) \tag{8}$$

$$Q = h\ A(T_b - T_w) \tag{9}$$

The water temperature rise was measured by the lateral displacement of a thermocouple junction from one section to the other. One sixteenth inch outside diameter stainless steel sheathed chromel-alumel thermocouple probes, with either insulated or exposed hot junctions, were employed with the ¼-in. cylinders, while a 1/25-in. O.D. insulated probe was used with the ⅛-in. O.D. circular cylinder. The temperature of the tube wall was determined for a variety of water flow rates with a thermocouple soldered to a hole in the wall.

Three inch long turbulence generators (springs or spirals) were wrapped around them to promote mixing of the fluid before it came in contact with the hot junction. It was assumed that the measured temperature was equal to the mean temperature across the section. Even if this were not so, it would be reasonable to expect that the temperature difference between the two cross sections would correspond to the real temperature difference because of the identical hydrodynamic conditions at both locations. Large fluctuations were observed in the thermocouple output whose amplitude ranged between 10 to 20% of the temperature difference. After investigation of their origin, it was found that they were mainly due to the flame instability, although imperfect mixing would account for part of them at too low a water flow rate. To obtain the

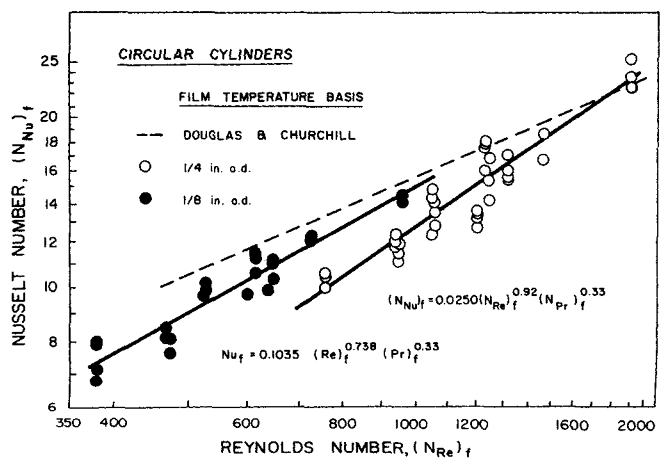

Fig. 2. Heat transfer results for ¼- and ⅛-in. O.D. circular cylinders. Properties at film temperature.

best temporal average of the water temperature, an integrating system was employed which consisted of a voltage-to-frequency converter and an electronic frequency counter. The mean temperature could be measured with a resolution of 0.05°C. for periods of 1 or 10 sec.

PROCEDURE

Eleven sets of operating conditions were selected, based on nozzle size, net power input to the gas in kilowatts, and gas flow rate in standard cubic feet per hour. These are listed in Table 1 with their identification number set and with their respective temperature and velocity results. To avoid radiation effects from the progressively hotter graphite wall, runs were conducted over short periods of less than 4 min. Hence, the wall temperature at the end of a run never exceeded 400°F.

Because the temperature and velocity profiles around the axis were relatively flat, temperature and velocity measurements were conducted simultaneously, each probe being 1/16 in. from the jet axis. Readings were taken every 15 sec.

For heat transfer rate measurements, the water flow rate was chosen so as to produce an appreciable temperature difference, about 4°F. The water temperature at each cross section of the test region of the cylinder was obtained by lateral displacement of the tube thermocouple assembly. After the system had been positioned, the new steady state was reached within from 4 to 5 sec. The water flow rate was determined by duplicate measurements of the time required to fill up a graduated 1-liter cylinder.

When heat transfer experiments for a cylinder were completed, the stability of plasma flame generation was checked by means of temperature measurements of the nitrogen stream. Because slight changes took place in the temperature and velocity of the jet as the chamber was heated up, all the measured quantities were determined from a plot of these quantities vs. time and evaluated 3 min. from start-up of the plasma jet.

STATE AND PHYSICAL PROPERTIES OF THE GAS

In the present system, the nitrogen stream experienced very large temperature changes. In the case of the highest enthalpy level run (3-23.4-150), the temperature and velocity dropped from about 10,000°F. and 2,500 ft./sec. to 4,300°F. and 110 ft./sec. along the axis of the jet, between the nozzle and the test section. Thus, the rate of temperature change was of the order of 10^6 °F./sec. Such a rate of change approaches the lower end of the range of temperature changes in which heterogeneous nonequilibrium systems are formed and in which some molecules are excited to much higher energy levels than others (*51*). However, Kubanek and Gauvin (*48*) calculated that the nitrogen molecules issuing from the nozzle of the torch would undergo about 10^6 collisions before reaching the test section. According to Greene et al. (*52*) and Blackman (*53*), molecular nitrogen required about 20 and 10^4 to 10^5 collisions for rotational and vibrational relaxation, respectively. Hence, it is reasonable to expect the gas at the test section to be in a state close to thermal equilibrium, since the requirements for it are almost satisfied for the most severe temperature change.

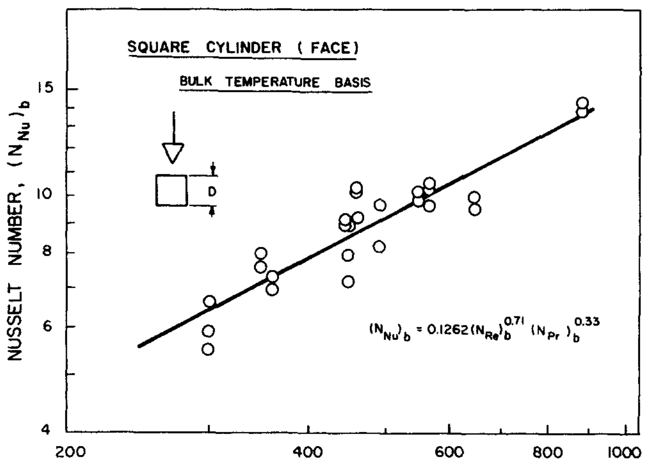

Fig. 3. Heat transfer results for ¼ in. square cylinder with a face perpendicular to the jet. Properties at bulk temperature.

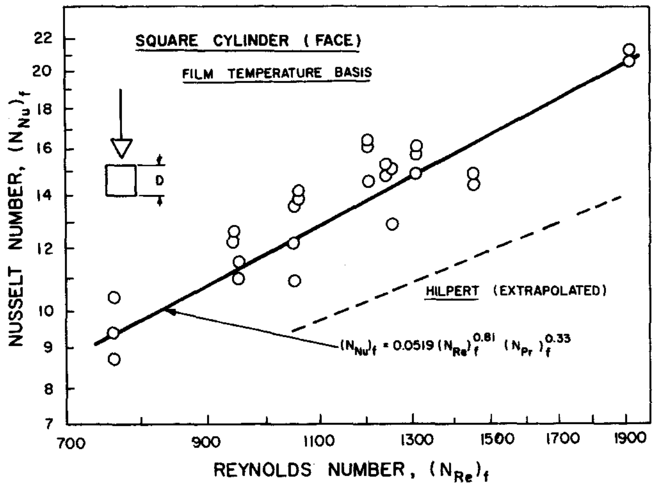

Fig. 4. Heat transfer results for ¼ in. square cylinder with a face perpendicular to the jet. Properties at film temperature.

For temperatures of 10,000° (at the nozzle exit) and 4,300°F. (at the test section), the equilibrium composition between the species, as given by Drellishak et al. (*54*), shows that the plasma at the nozzle, if at equilibrium, is in a partly molecular (70%) and partly dissociated (30%) form, while the gas is in a quasimolecular state at the test section. Hence, the physical properties used at the test section were assumed to be those of molecular nitrogen at atmospheric pressure as proposed by John et al. (*55*) for the viscosity, references 55 to 57 for the thermal conductivity, and Drellishak et al. (*54*) for the density and the specific heat at constant pressure.

RESULTS

Velocity and Temperature Measurements

Average temperature and velocity of the jet for each operating condition are given in Table 1. Temperatures and velocities were found to be reproducible with a maximum deviation of 8 and 5%, respectively. Temperature fluctuations of at least 100°F. were detected in the jet.

Heat Transfer Measurements

Heat transfer data are reported in the Nusselt number form, with both the bulk and film temperatures as reference temperatures, on the basis of the equation

$$N_{Nu} = (D/k)\ [Mc_p\ (T_{out} - T_{in})]/[A\ (T_b - T_w)] \qquad (10)$$

Plots of the results are presented in Figures 1 and 2 for the circular cylinders, Figures 3 and 4 for the square cylinder face, and Figures 5 and 6 for the square cylinder

Table 1. Temperature and Velocity Results at Designated Plasma Torch Operating Conditions

Nozzle No.	Net power input to gas, kw	Nitrogen flow, std.cu.ft./hr.	Run designation	T_b, °F.	U_b, ft./sec.
1	11.0	100	1-11-100	2,260	77.7
1	11.0	150	1-11-150	1,920	85.4
3	11.0	150	3-11-150	2,540	64.4
3	11.0	200	3-11-200	2,210	63.9
3	14.0	100	3-14-100	3,440	66.1
3	14.0	150	3-14-150	2,820	73.3
3	14.0	200	3-14-200	2,550	80.0
3	19.5	150	3-19.5-150	3,900	96.7
3	19.5	200	3-19.5-200	3,390	105.8
3	23.5	150	3-23.5-150	4,270	108.7
3	23.5	200	3-23.5-200	3,840	121.7

edge.* The maximum deviations in heat flux rate were found to be about ± 10%. For circular cylinders, comparisons of the present results with those of Churchill and Brier (*25*) and Douglas and Churchill (*34*) are given in Figures 1 and 2, respectively. For the square cylinders, the line of Hilpert (*16*) has been extrapolated down to the N_{Re} range studied (the lower limit of the range investigated by this author was 5,000).

A comparison of the heat transfer correlations for the different geometries [including the results for spheres obtained by Kubanek et al. (*1*)] is presented in Figure 7. It was ascertained that conduction effects along the wall of the cylinders, as well as natural convection effects, were unimportant compared with the forced convective heat transfer.

Treatment of the Data

Two relationships have been tried to fit the data. The first model was

$$N_{Nu} = a\ N_{Re}^{n} \qquad (11)$$

while in the second a temperature ratio (T_b/T_w) was included to account for the large temperature difference and its effect on the physical properties:

$$N_{Nu} = a'\ N_{Re}^{n'}\ (T_b/T_w)^{n} \qquad (12)$$

Results of the least-square analysis by computer are presented in Table 2.

The Prandtl number, being practically constant $(0.69 < N_{Pr} < 0.71)$, was not considered during the processing of the data but was included at the end to generalize the correlations. The temperature of the wall was in the narrow range 120° to 140°F., and an average value of 130°F. was used in all the calculations.

DISCUSSION OF RESULTS

Table 2 clearly shows that the exponent of the Reynolds number is greater than 0.5, which is interpreted as indicating that a part of the attached boundary layer is turbulent. According to Torobin and Gauvin (*41*), transition in the boundary layer will take place even at low Reynolds numbers if the turbulent energy of the free stream is large enough (compared with the viscous damping energy in the initially laminar boundary layer) to allow sufficient penetration of the disturbances into the boundary layer. It seems that in the present study transition to a turbulent boundary layer was attained at various distances downstream from the stagnation point (as reflected by the varying values of n reported in Table 2) owing to the following considerations:

1. There is a very high turbulence level of the jet generated by the torch. According to references *58* and *59*, the intensity of turbulence of a confined jet is about 20 to 25% at a distance of about 40 diam. from the nozzle.
2. The cyclic process of the arc generates pockets of plasma about 1 in. long with a diameter of 0.2 to 0.3 in. (*1*). Owing to the large axial and radial velocity gradients, these pockets will exchange energy by turbulent diffusion and decrease in size. However, this phenomenon enhances the turbulence of the jet.
3. The important decrease in the value of the nitrogen viscosity through the boundary layer towards the wall will decrease its stability towards disturbances in the vicinity of the latter (μ can be two to three times smaller at the wall than in the jet).

Before we discuss the results in more detail, some considerations are given now concerning model No. 2.

Significance of the Temperature Ratio

Table 2 shows that the temperature ratio (T_b/T_w) does not make a consistent contribution to the heat transfer. However, it can be noted that large temperature differences seem to have a tendency to decrease the value of the heat transfer coefficient, as indicated by the negative value of most of the exponents. The only exception occurs in the case of the square cylinder results, which are more scattered than the others (due to a possible yaw in the support when heated).

Although no significant conclusions can be drawn from Table 2, it can be remarked that such an effect is coherent. The effects of a large temperature difference can be expected to be reversed for the heating and the cooling of a surface by a gas because the temperature and velocity profiles react to large ΔT in opposite ways when the direction of the heat flow is changed. The careful investigations of Hilpert (*16*) and Collins and Williams (*60*) have shown that large ΔT's increase the heat transfer for the case of the cooling of a cylinder. Hence a decrease of the heat transfer coefficient can be expected for the heating of a cylinder by a hot gas, as in the present study. It should be noted that Churchill and Brier (*25*) found the reverse effect in the latter case. However, the inclusion of the temperature ratio in their relationship is not warranted,

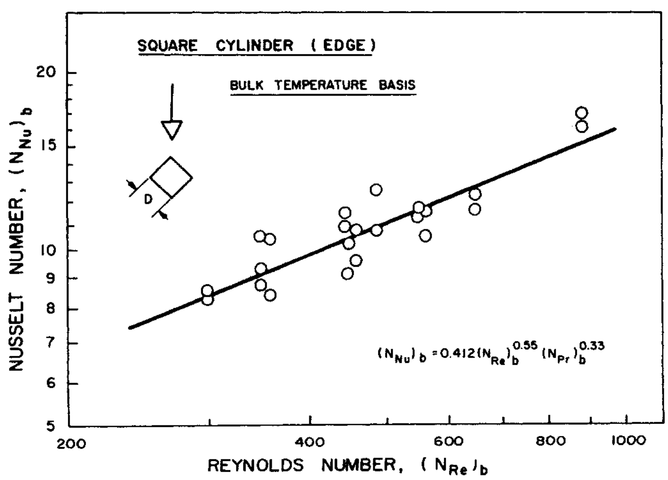

Fig. 5. Heat transfer results for ¼ in. square cylinder with an edge facing the jet. Properties at bulk temperature.

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TABLE 2. PROCESSING OF THE DATA FOR MODEL NO. 1 ($N_{Nu} = aN_{Re}^n$) AND MODEL NO. 2 [$N_{Nu} = a'N_{Re}^{n'} (T_b/T_w)^m$]

Shape	Model and ref., T	a or a'	Standard error ($\log a$)	n or n'	Standard error (n) or (n')	m	Standard error (m)	Degree of freedom	Standard error, ($\log y$)	Corr. coeff., R
¼-in.	$1 - T_b$	0.054	0.269	0.836	0.043			33	0.072	0.958
O.D.	$2 - T_b$	0.172		0.708	0.069	−0.204	0.088	32	0.068	0.964
circular	$1 - T_f$	0.022	0.439	0.910	0.061			33	0.083	0.929
cylinder	$2 - T_f$	0.186		0.698	0.066	−0.324	0.068	32	0.064	0.961
⅛-in.	$1 - T_b$	0.147	0.183	0.690	0.033			21	0.051	0.976
O.D.	$2 - T_b$	0.367		0.582	0.059	−0.187	0.086	20	0.047	0.980
circular	$1 - T_f$	0.092	0.312	0.738	0.049			21	0.061	0.956
cylinder	$2 - T_f$	0.458		0.560	0.059	−0.281	0.071	20	0.047	0.976
Square	$1 - T_b$	0.112	0.408	0.710	0.066			24	0.095	0.909
cylinder	$2 - T_b$	0.0197		0.901	0.085	0.326	0.103	23	0.082	0.936
face on	$1 - T_f$	0.0459	0.517	0.809	0.073			24	0.087	0.914
	$2 - T_f$			Same as model No. 1						
Square	$1 - T_b$	0.365	0.377	0.550	0.060			21	0.082	0.892
cylinder	$2 - T_b$			Same as model No. 1						
edge on	$1 - T_f$	0.268	0.576	0.603	0.071			21	0.079	0.880
	$2 - T_f$			Same as model No. 1						
Spheres	$1 - T_b$	0.104		0.756	0.016			57	0.061	0.987
	$2 - T_b$			Same as model No. 1						
	$1 - T_f$	0.048		0.822	0.021			57	0.068	0.982
	$2 - T_f$			0.749	0.033	−0.119	0.43	56	0.064	0.984

as indicated by the low correlation coefficient (less than 0.2) reported by Kubanek (*61*) after he tested the significance of this ratio in their correlation. This may be due to the condition imposed on the Reynolds number exponent, which was fixed by Churchill and Brier at the value of 0.5.

Although the introduction of the temperature ratio may improve the correlations very slightly in the present investigation, particularly in the case of data evaluated on the film temperature basis, its significance is doubtful, and model No. 2 was discarded. The correlation of the data by Equation (11) is therefore adopted for the purpose of the rest of this discussion. Although the value of n was particularly affected by the experimental points at each end of the Reynolds number range investigated, its value for each geometry is consistent with the expected flow configuration.

Circular Cylinders

Figures 1 and 2 show the results for the ¼- and ⅛-in. O.D. circular cylinders fall on two lines. This was confirmed by a statistical test by the computer. Owing to a lack of information about the turbulence of the jet, and more particularly about its scale, no argument is available to explain this discrepancy. Nevertheless, a possible effect of the combined action of the scale of turbulence and of the initial pockets of plasma can be suggested, the diameter of the ⅛-in. O.D. cylinder being small enough. It should be remembered than Van der Hegge Zijnen (*14*) has recommended the use of a (L_x/D) ratio, but this suggestion has never been vindicated. In addition, it should be pointed out that the results for the ¼-in. O.D. cylinder are more reliable from an experimental point of view, as well as much more plentiful, and the preferential use of the latter results is recommended. From the high value of the exponent n on the Reynolds number, it can be inferred that the attached boundary layer is turbulent over most of its course, and that the contribution of the small wake to heat transfer (the point of separation is well in the rear of the cylinder) is not very important.

Comparison with the data of Douglas and Churchill and of Churchill and Brier shows that the present data are

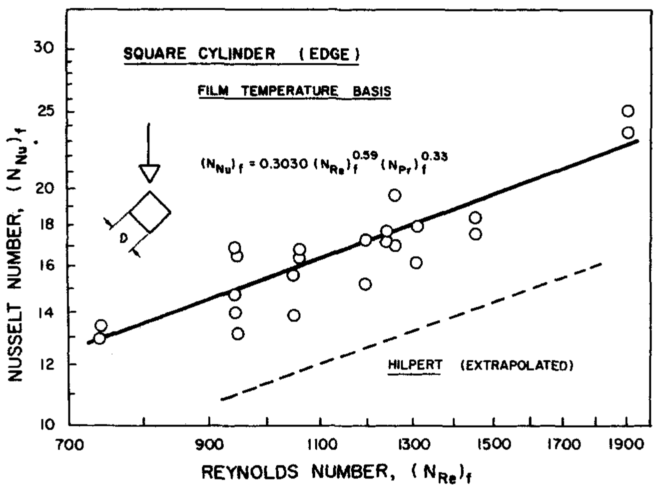

Fig. 6. Heat transfer results for ¼ in. square cylinder with an edge facing the jet. Properties at film temperature.

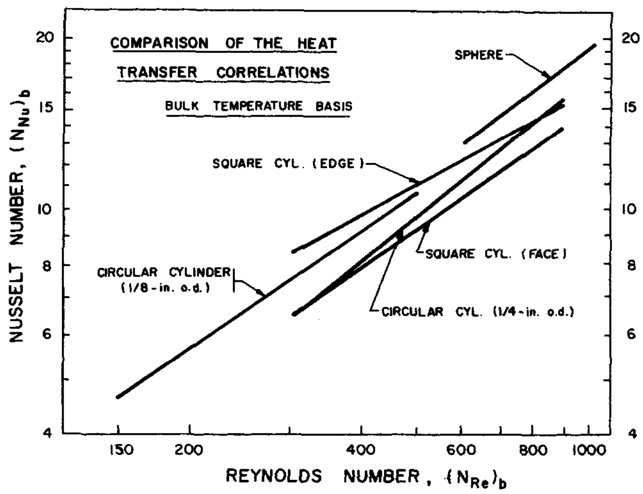

Fig. 7. Comparison of the heat transfer correlations. Bulk temperature basis.

slightly below their results. This can be attributed to the difference in the temperature levels at which the experiments were conducted (the variations of the physical properties are much larger in the present investigation) and mainly to the essential difference in the boundary-layer regime.

Square Cylinder with a Face Perpendicular to the Jet

This case suggests the existence of a turbulent boundary layer as well. The essential difference with the previous case is that a separation occurs as the upstream corners, followed in all likelihood by reattachment of the boundary layer. Final separation then takes place at the edges of the lower face. Owing to the fixed separation of the boundary layer, the wake contribution in this case (which by itself would have a value of n of 2/3) is larger than for a circular cylinder. The relative scatter of the data for this geometry may be due to a certain yaw in the tube position.

Square Cylinder with an Edge Facing the Jet

In this geometry (which is roughly similar to a wedge body), separation points will be located at each side of the mid cross-section plane, and therefore the wake contribution will occupy 50% of the total surface area. The value of n in this case seems to indicate that the attached boundary layer was essentially laminar. The rate of change of the pressure gradient in the front part of the body being less for this configuration (as in the case of a wedge), it is conceivable that transition, if it occurs, will take place further downstream than in the previous cases.

The quasilaminar boundary-layer region and the wake region being well delimited, a heat transfer relationship of the type proposed by Richardson (*13*) was attempted, namely

$$N_{Nu} = C_1 \ N_{Re}^{1/2} + C_2 \ N_{Re}^{2/3} \tag{13}$$

However, processing of the data revealed that the correlation coefficient was low (less than 0.3), and this model was therefore given up.

CONCLUSIONS

Based on the data obtained, the following correlations are recommended for the various geometries studied [the results of Kubanek et al. (*1*) obtained on spheres are included for comparison], with the physical properties evaluated at bulk temperature:

Sphere

$$N_{Nu} = 0.118 \ N_{Re}^{0.76} \ N_{Pr}^{0.33} \tag{14}$$

Circular cylinder

$$N_{Nu} = 0.0615 \ N_{Re}^{0.84} \ N_{Pr}^{0.33} \tag{15}$$

Square cylinder with a face perpendicular to the jet

$$N_{Nu} = 0.126 \ N_{Re}^{0.71} \ N_{Pr}^{0.33} \tag{16}$$

Square cylinder with an edge facing the jet

$$N_{Nu} = 0.412 \ N_{Re}^{0.55} \ N_{Pr}^{0.33} \tag{17}$$

The characteristic length in the last two cases is the side of the square.

It must be emphasized that these correlations apply only to systems of high intensity of turbulence, as studied in the present investigation. As a result, it can be inferred that the initially laminar boundary layer will undergo transition at various points downstream of the front stagnation point, depending on the different pressure gradients imposed by the different geometries studied. In other words, because of the form of the above equations, the value of n will reflect the combined contributions from the attached boundary layer in the front and the wake region in the back. Thus, a logical increase in n was obtained from a wedge type of body with a large fixed wake (square cylinder edge), through a configuration yielding a smaller wake and a larger turbulent boundary-layer repartition (square cylinder face), and finally to configurations with small wakes and maximum extent of turbulent boundary layers (spheres and circular cylinders).

Under these conditions, it is not surprising that a better agreement is obtained by estimating the physical properties of the fluid at the bulk temperature. For most of the published correlations of data on heat transfer to bodies of various shapes, the mean fluid temperature has been reported to give better results. Since these studies were invariably correlated by means of $N_{Re}^{0.5}$, there is little doubt that the boundary layer was in a laminar regime, and an arithmetic average temperature would be expected to represent a good average for the estimation of the physical properties. When the boundary layer is turbulent, however, the intense mixing action in this region should be expected to yield an average temperature in the layer much closer to that of the free stream.

Previous studies with the aim of finding a standard characteristic length to correlate the convective heat transfer to bodies of various shapes were found to be fairly successful, because for all of them the attached boundary layer was in an essentially laminar regime (*43* to *46*). Therefore, it was possible to define a characteristic length which could reasonably account for the difference in the flow pattern around the various geometries, even for fairly severe conditions of free stream turbulence (10% in the case of reference *44*), providing that transition to a turbulent boundary layer did not occur. The present study, however, clearly shows that the concept of a universal characteristic length breaks down as soon as mixed regimes in the boundary layer are involved, because a similarity in the transition of the layer does not exist for the various bodies.

Although the model proposed by Richardson (*13*) was not successful in correlating the results obtained for a cylinder with an edge facing the jet, it is felt on the other hand that the form of the Equations (14) to (17) is oversimplified and that a more complex relation, involving the separate contributions from the attached boundary layer and from the wake region (with possibly an additional interaction term), would be more successful in predicting the heat transfer rates corresponding to various geometries. Similarly, the effects of various levels of turbulence intensity should be accounted for in these equations. Much more theoretical knowledge and experimental data must, however, be acquired before a better understanding of the complex mechanism of heat transfer can be obtained.

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NOTATION

a, a' = constants
A = lateral surface area, sq.ft.
c_p = specific heat at constant pressure, B.t.u./(lb.)(°F.)
C_{A^+}, C_{N^+}, C_N = concentration of A^+ ions, N^+ ions, N atoms, respectively, ft.$^{-3}$
D = diameter, ft.
h = heat transfer coefficient, B.t.u./(hr.)(sq.ft.)(°F.)

I = intensity of turbulence, $[((u')^2)^{0.5}/U]$
k = thermal conductivity, B.t.u./(hr.)(ft.)(°F.)
L' L'' = characteristic lengths, ft.
L_x = Eulerian macroscale of turbulence, ft.
M = water flow rate, lb./hr.
m, n, n' = exponents
N_M = Mach number
N_{Nu} = Nusselt number, (hD/k)
N_{Pr} = Prandtl number, $(c_p\mu/k)$
N_{Re} = Reynolds number, $(DU\rho/\mu)$
p_m = mid cross-section perimeter, ft.
Q = heat transfer rate, B.t.u./hr.
R = correlation coefficient
T = temperature, °F. or °R.
u' = fluctuating component of the velocity, ft./sec.
U = velocity, ft./sec.
W = duct width, ft.

Λ = Taylor parameter
π = 3.14159
μ = viscosity, lb./(ft.)(hr.)
ρ = density, lb./cu.ft.

Subscripts

aw = adiabatic wall
b = bulk
c = lower critical
f = film
m = mean integrated
t = total
w = wall

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